Voices of Mathematicians and Mathematics Teacher Educators Co-Teaching a Mathematics Course for Prospective Secondary Teachers

Both mathematicians and mathematics teacher educators (MTEs) have responsibility for preparing preservice mathematics teachers (PSTs). In many institutions, mathematics content courses are taught by mathematicians, and mathematics pedagogy courses by MTEs. In separate departments or colleges, these two groups often live in different worlds with different cultural norms. The Conference Board of the Mathematical Sciences (CBMS, 2001) notes: "There is considerable distrust between mathematics faculty and mathematics education faculty both within institutions and through public exchange. Conscious efforts … are needed to foster cooperation, along with mutual understanding and respect" (p. 9).

Public discourse about mathematics teacher preparation is often based on content knowledge. Yet as Ball (2003) acknowledges, "increasing the quantity of teachers' mathematics coursework will only improve the quality of mathematics teaching if teachers learn mathematics in ways that make a difference for the skill with which they are able to do their work." Engaging teachers in learning mathematics in ways that resonate with future expectations for teaching suggests a need for collaboration between mathematicians and MTEs as recommended by the CBMS:

> Most good school mathematics instruction involves a combination of mathematical knowledge and pedagogy …. Mathematics educators can provide valuable insights and information about what takes place in school classrooms, including common mathematical misunderstandings of practicing teachers. … [M]athematics faculty can help mathematics education faculty by keeping them informed of mathematical developments which have an impact on school mathematics. (p. 9)

Collaboration can take many forms, from sharing of ideas and philosophies in occasional discussions (Ball et al., 2005) to developing curriculum materials for professional development with teachers (Kersaint & Berger, under review) to co-teaching of courses (Grassl & Mingus, 2007). In this paper, we share our experiences in creating a learning community from a collaboration among a mathematician (Catherine), two MTEs (Denisse and Gladis), and a

the Fall 2009 semester.

Neither Catherine nor Denisse had ever previously taught this college-level geometry course nor had they previously collaborated in any way. The course met 75 minutes twice each week. Individual desks were arranged in rows, and a computer smart system and document camera were permanently in the room.

Overall Goals of the Course and Collaboration

When we first met to design a syllabus, we established some objectives that would permeate the course. Specifically, PSTs should

> learn mathematics using inquiry-based approaches as recommended by the mathematics education community (e.g., Martin, 2007;

instructional roles in each class period, we designated in our notes who would lead each segment of the lecture or class activity. Thus, we created an expectation for ourselves that Denisse and Catherine had a significant contribution to make during every class

A Look into the Class: Typical Activities

First Day. Setting the tone for the semester on the first day of class was crucial. We wanted to establish both for ourselves and for the PSTs that mathematics as well as education issues would be present and that a social environment would be created in which PSTs would discuss the content and work with each other. So, we began this class by having the PSTs put themselves in order according to their birthdays (day and month) without speaking. Such an activity strongly hinted that this class might be different from what they normally experienced in mathematics! We had the PSTs discuss why we had engaged in such an exercise, and they raised issues of communication, group work, and their own expectations for a mathematics class. The activity provided a shared experience for all to serve as a foundation for what would occur throughout the semester.

had read prior to class about important lines in a triangle (i.e., median, altitude, angle bisector).

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papers and made any adjustments before returning assignments. This shared responsibility for grading gave each instructor ownership of the course and was key to the collaboration being significant. PSTs generally could not tell who graded their assignments.

Sharing our perspectives on the collaboration

In this section, we share our individual perspectives on the collaboration. We start with the two co-instructors (Catherine, the instructor of record, and Denisse), followed by the two mathematics educators who observed the class (Gladis and Sarah).

Catherine speaks as a mathematician co-teaching the course. I had never team taught a course before; sharing the privacy and intellectual domain of the classroom was initially difficult. Having observers (Gladis and Sarah) made me particularly self-conscious, and I had to adjust to receiving constructive criticism. Because I had not taught geometry before, I had few pre-conceived notions about the course itself and was willing to keep an open mind about topics to cover or strategies to try. After a few weeks, I began to enjoy the natural cycle of feedback and discussion about what went well and what didn't. One day when Denisse couldn't make it to class and the others were late, I was disappointed to be on my own; I had become accustomed to sharing the classroom and bouncing ideas off her during class.

One of the things I most enjoyed about the collaboration was the preparation for class. We each reviewed the mathematics in the sections we planned to cover before our meeting, shared ideas about what concepts were important, and brainstormed about how they should be taught and the activities we might incorporate. We spent time working challenging geometry problems. Because we were using a traditional mathematics text, we struggled to identify ways of teaching the material that would support an inquiry approach. I found the mathematical and pedagogical challenges of the preparation intellectually stimulating.

Before I began this collaboration, I considered myself a "good" teacher and someone who connects to students and recognizes what they do not understand. I had previously experimented with various strategies (e.g., group work, technology), but I feel that there was something fundamentally different about the initial setup of this geometry class. Although the strategies we employed were not that different from those I had implemented on occasion in other classes, they were incorporated as a fundamental piece of the geometry course design.

This collaboration has given me numerous concrete ideas about how to deliver instruction with a student-centered approach, such as having students use patty paper in geometric constructions or spaghetti to discover the triangle inequality. One specific technique I found useful was to create a handout of students' quiz responses for them to critique. My role shifted to discussion leader, and criticisms came from students, who were able to judge, with a little guidance, how what was written was not coherent or correct, or needed some adjustment. Through such activities, students became more critical during the semester, better able to evaluate peers' responses, and more adept at writing their own responses in future assignments. In addition, Denisse and I gained access to what the students really understood. It became clear that students were having great difficulty in identifying the hypothesis and conclusion of a given mathematical statement, and we needed to spend time helping them understand references to any pronouns in the statement of a theorem before we had any hope of teaching students to prove it.

I also became familiar with what students themselves will face in the classroom in terms of content, and more importantly, I gained a sense from my collaborators about what is emphasized in the high school curriculum, such as the use of congruence tests for triangles. This is difficult for a mathematician to know if working alone.

Throughout the semester, I became more aware of how my own imprecise use of notation or

I completed my mathematics coursework well before the implementation of the standards movement from NCTM. Even though I try to teach mathematics pedagogy courses with discussions, cooperative group work, and problem solving, I have not had an opportunity to implement those practices into a mathematics content class. I was faced with the practical reality that many of the recommended pedagogical approaches are challenging to implement, especially when you feel that so much content needs to be addressed. Deciding that it is okay to give

with students, this was not what I had expected from the mathematicians at my university.

I feel I was an equal partner in teaching the geometry course, but the time commitment was huge. I was not technically listed as an instructor of record. Because we were participating in the grant, I had a course release during the year. But without such a release, could I afford to spend the time that such collaboration requires? In mathematics education, we teach the same courses on a regular basis so time commitments for intense reflection and course modification pay off. In contrast, in our mathematics department, courses circulate among various faculty members. So, I could invest the time to suggest improvements in one of the content courses taken by our PSTs and it might be for naught when someone else teaches the course in the next semester. Regardless of the outcome, I learned that it is important for MTEs to take an active role in enhancing mathematic content courses, particularly those taken by prospective teachers.

Gladis speaks as a mathematics teacher educator observing the course. Unlike Catherine and Denisse, I spent several years teaching high school geometry using many of the strategies recommended for instruction by the mathematics education community, including the use of technology. So, I was looking forward to my role as a co-instructor and to incorporating those strategies in teaching a university content course. Although I couldn't be a co-instructor, I thought it would still be possible to incorporate many of those approaches through my participation in the planning. However I found this to be a challenge.

Because I was familiar with all the players, I assumed trust would exist from the onset. Denisse and I had collaborated in various capacities, including co-teaching a course, co-authoring manuscripts, and collaborating on programmatic issues. Catherine had been involved in a prior grant effort to develop and deliver content-specific professional development which resulted in my observing her teaching. Sarah was a doctoral student in one of my courses.

goal?" and "How do you see this playing out in the class?" Responses to these questions revealed differences in interpretations, providing opportunities to clarify understandings.

I am pleased with this collaboration. The classroom environment, students' feedback, and collaborator feedback have been positive. This collaboration has reinforced previous work in engaging mathematicians meaningfully in the work of teacher education. There is greater appreciation of the role both groups play. These initial efforts will broaden the discussion among MTEs and mathematicians and build the foundation for other efforts to improve the

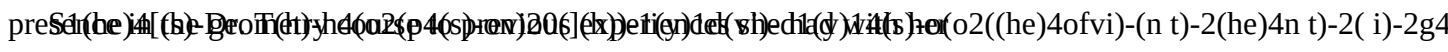
within mathematics courses. As a student new to the field of education myself, and accustomed to the traditional teacher-centered, lecture-style instruction within the mathematics department, I often feel overwhelmed by the recommendations for inquiry-based teaching and learning. What exactly does it mean to teach in an inquiry-based fashion? Had I ever seen a teacher who taught that way? What would it look like? I can't help but believe PSTs ask themselves the same questions. Within this course, PSTs were able to experience first-hand an inquiry-based classroom. For example, after working on a problem collaboratively in small groups, Catherine and Denisse would pull the PSTs back together for a whole-class discussion in which the PSTs' contributions, as opposed to the instructors', would determine the flow and direction of the discussion. PSTs listened closely to each other, focused on the accuracy and precision of mathematical language used within the classroom, and questioned each other when something was unclear or seemingly incorrect.

I believe this type of classroom environment was successfully cultivated as a direct result of our collaboration. Our differing levels of experience as teachers, mathematicians, and teacher educators contributed to the variety of perspectives through which we viewed the course. As MTEs, Gladis and Denisse identified activities that would help PSTs discover mathematical relationships on their own and develop precision of mathematical language. As a mathematician, Catherine recognized connections among foundational aspects of the subject that helped lead the PSTs derive formulas instead of simply memorizing them (e.g., the law of cosines). As someone new to education, I was eager to learn about pedagogical tools and activities teachers could use in their own classrooms, and therefore researched and proposed several such activities (e.g., breaking up spaghetti into three pieces to formulate a conjecture about the triangle inequality). While Catherine and Denisse were teaching, Gladis and I made observations and discussed the

unfolding events. We viewed the instruction from the perspective of the instructors and also focused our attention on the reactions of the PSTs to the instruction. During our weekly meetings, the four of us discussed class sessions from our different perspectives and used these reflections to design the next class accordingly.

I gleaned several personal lessons from participating in this collaboration. Through interaction with and observation of secondary PSTs, I made valuable connections between research and practice. Because my current research interests focus on the teaching and learning of mathematical proof, being in the classroom with PSTs and reflecting on their learning of proof in geometry helped bring readings from research to life.

Prior to this collaboration, I would have resisted having someone observe my classroom. But now, I



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